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USING THE INTERNAL RATE OF RETURN IN PUBLIC SECTOR PROJECT EVALUATIONS

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Abstract — Project evaluation in the public sector is a recent development, to which there are basically two approaches. One is to capitalize the flow of future costs and benefits as net present value (NPV) and to prefer the higher NPV when choosing between two projects. The other is to find the internal rate of return (IRR), that is, the discount rate that brings the flow of costs in equilibrium with the flow of the benefits at a particular time. The tendency has been to prefer the NPV method for its ease of calculation, but this need no longer concern us, given the widespread availability of personal computers. In addition, IRR was alleged to be ambiguous; again this is easily remedied. The IRR has the advantage that it avoids the need to choose a discount rate, which involves a risk of serious bias.

Résumé — Dans le secteur public, l'évaluation de projet représente un développement récent. On distingue fondamentalement deux approches. L'une d'elles consiste à capitaliser la succession des dépenses et des profits futurs comme 'valeur actuelle nette,' et de donner la préférence à la valeur actuelle nette la plus élevée lorsque l'on choisit entre deux projets. La solution alternative consiste à déterminer le 'taux de rendement interne', à savoir le taux d'actualisation qui amène la succession des dépenses et celle des profits au même niveau à un moment précis. On a eu tendance à préférer la méthode de la 'valeur actuelle nette'. Cette méthode était plus facile à calculer, mais ceci importe peu de nos jours étant donné la disponibilité des ordinateurs personnels, et l'ambiguïté supposée de la méthode du 'taux de rendement interne' est aisément évitée. La méthode du 'taux de rendement interne' offre l'avantage d'éviter la nécessité de choisir un taux d'actualisation où le risque d'un grave manque d'objectivité existe.

THIS ARTICLE OFFERS A re-evaluation of the *internal rate of return* (IRR) as a method for conducting project evaluations in the public sector. The approach that has been most widely used in public sector project evaluation has been the *net present value* (NPV) method. The advent of the personal computer has transformed the tedious calculation of IRR into a simple routine, and so has shifted the balance of advantage, which formerly has favored the NPV method.

It is useful to begin with the purely financial model developed for standard financial contracts such as mortgages and annuities, and then to consider how the mathematical model derived from financial transactions was extended to analyses of the real return on capital investment in the private sector. Finally, I consider the implications of extending this model into the public sector, where social costs and benefits have to be taken into account even if they are not reflected in market transactions.

THE FINANCIAL MODEL

The basis of the financial transaction is interest, understood as the rental charge for the use of money. If money is left with a financial institution, with interest credited at regular intervals, the principal on which future interest is calculated will be increased by the interest accumulated. This leads to the concept of compound interest. A sum of \$100 at 5% interest will amount to \$105 after one year, \$110.25 after two years, \$115.7625 after three years, and so on. After n years the principal (P) will amount to A_n at the end of n years at i% interest compounded annually:

$$P(1+i)^n = A_n$$

The equation can be rearranged to give the present value (P) of an amount A in n years discounted at i%:

$$P = \frac{A_n}{(1+i)^n}$$

An amount of \$100 in three years' time at 5% has a present value of:

$$\frac{100}{(1+0.05)^3} = 86.384$$

More important for our purposes, if the principal P and its future value A_n are known, it is possible to calculate the implied rate of interest (i)[1] on the money invested:

$$i = \left[\frac{A_n}{P}\right]^{1/n} - 1$$

From these relationships come a number of standard financial transactions that use interest or discount rates to relate a capital sum and a stream of recurrent payments over a given period of time (e.g. annuities, mortgages, sums assured). These calculations all depend on the concept of compound interest, the rate of which may be given, or can be calculated if all the other elements are known.

No assumption is made as to why those making the loans choose to save rather than to spend on current consumption. It is enough that there are surpluses and that these will be made available, other things being equal, to the borrower offering the highest interest rate. The interest rate therefore serves as a rationing device, where potential borrowers exceed the supply of funds at zero interest. The calculations are all done in terms of money, and the question of changes in the purchasing power of money is not considered.

REAL CAPITAL FORMATION: THE FINANCIAL MODEL EXTENDED TO REAL ASSETS WITH REAL RETURNS

Capital formation involves using real resources, that might have been spent on current consumption, to produce capital goods that will contribute to a higher level of future production. The increase in future production must be sufficient to repay or replace the resources devoted to capital production and to justify the postponed consumption.

The variables in the calculation are K, the capital cost; a_i, the revenue minus the costs in year i for all values of i from $i = 1$ to $i = n$, where n is the life of the project in years; and r, the annual rate of return on the investment.[2] In such a transaction, interest is not explicitly involved, although it is possible, given some assumptions about the economic life of the asset, to calculate the implied rate of return (r) by solving the equation:

$$K = \frac{a_1}{(1+r)} + \frac{a_2}{(1+r)^2} + \dots\dots\dots + \frac{a_n}{(1+r)^n} = \sum_{i=1}^{n} \left[\frac{a_i}{(1+r)^i} \right]$$

This calculation is tiresome; fortunately, it is precisely the sort of task that computers handle so easily. The calculation of r gives a means of comparing projects with different expected lifespans or involving different capital sums. Simply choose the project with the highest r. It offers the highest return on capital. It also provides a basis for deciding whether it is worth borrowing at a given rate of interest in order to finance such capital formation, or whether it is more rewarding to sell the resources and lend the proceeds to earn interest.

In the case of real capital formation, we are interested in the return in future real earnings, or to rephrase, in the stream of benefits that the asset will produce. An individual or firm will benefit from borrowing money and paying interest to finance capital investment so long as the rate of interest is less than the rate of return. It follows that, even where there is an investment with an expected positive return, it will be more profitable to invest elsewhere if doing so will produce a higher expected rate of return.

In theory, market interest rates will tend to reflect the real return earned by capital, because would-be lenders can maximize their pay-off by seeking out those borrowers with projects that offer the best return on the capital invested. But financial markets are imperfect; they have difficulty in assessing risk, so it is not surprising that, in practice, market interest rates are not particularly good indicators of capital returns.

Social Cost-Benefit Analysis and Public Sector Project Appraisal

Social cost-benefit analysis extends the economic concepts used to evaluate private sector projects into the public sector, taking account of benefits and costs to society as a whole even where there are no corresponding cash flows, or where the cash flows do not adequately reflect the full social costs and benefits.[3] The considerable problems posed by attaching money values on these items have been discussed extensively in the literature. I do not propose to reiterate them here.

Capital projects, such as bridges or roads, have most often been the subjects of cost-benefit study. But the concept can be extended to the whole range of public expenditures, including the evaluation of programs that have recurrent rather than capital costs, such as antenatal treatment for women, and other forms of preventive medicine.

Public sector project appraisal has a number of objectives:

1. To distinguish sound from unsound investments. A project is unsound if the benefits do not cover the costs.

2. To choose between mutually exclusive projects or to rank projects in order of preference so that the most attractive projects take precedence when funds are limited. Budget constraint makes the problem one of constrained maximization, the guiding principle of which is: Maximize the return subject to a budget constraint.

ALTERNATIVE APPROACHES TO THE CALCULATION OF COST-BENEFIT ANALYSIS

There are two ways to tackle the arithmetic of evaluation: Calculate the *NPV* of the flow of costs and benefits, or calculate the *IRR*. The NPV method uses a given rate of discount to reduce all costs and benefits to a common time period, with the objective of maximizing the NPV attainable with a given budget. If the NPV is positive, the project meets Criterion 1.

To illustrate, suppose an asset can be leased for three years for a payment of $100,000, and it is possible to earn $50,000 for each of the three years. What is the return on the initial $100,000?

Solution 1, using NPV.

Assume the borrowing rate is 10%. Working in units of $1,000,

$$NPV = \frac{50}{1.1} + \frac{50}{1.1^2} + \frac{50}{1.1^3} - 100 = 24.3426$$

The project is worth $24,343 using a discount rate of 10%, but a higher rate would reduce that NPV. This project is worth undertaking in the sense that what it produces is worth more than what it costs, but it may not be the best use of $100,000.

The other approach is to calculate the rate of return (r), which is implicit in the costs and benefits. Using the same numerical example, what is the return on the initial $100,000? The general formula is:

$$\sum_{i=0}^{n} \left[\frac{R_i}{(1+r)^i} \right] = \sum_{i=0}^{n} \left[\frac{C_i}{(1+r)^i} \right]$$

where R_i is the revenue in year i; C_i is all cost in year i, including capital and recurrent costs; and n is the life of the project in years. The task is to find r, given the values for R_i, C_i, and n.

Solution 2, using IRR.

Again working in units of $1,000, the relevant equation is:

$$\frac{50}{1+r} + \frac{50}{(1+r)^2} + \frac{50}{(1+r)^3} = 100$$

To solve this equation for r, we try various values of r. If the left-hand side is less than the right-hand side, the value of r needs to be decreased; if the converse, r should be increased.

Try with *r*	=	25%	97.6	<	100
Decrease *r*					
With *r*	=	23%	100.5687	>	100
Increase *r*					
With *r*	=	23.35%	100.038	>	100

This may be close enough to equality for practical purposes, in which case we can say the project yields slightly more than 23.35%. To be viable (i.e., to meet Criterion 1), the value of *r* must be positive. In order to choose between two or more projects (as Criterion 2 requires), one need only repeat the calculation of *r* for the other projects, and choose the one with the highest value of *r*. In most cases, the *NPV* method and the *IRR* method yield the same answer, but a problem arises when the two methods give different preferences.

Figure 1

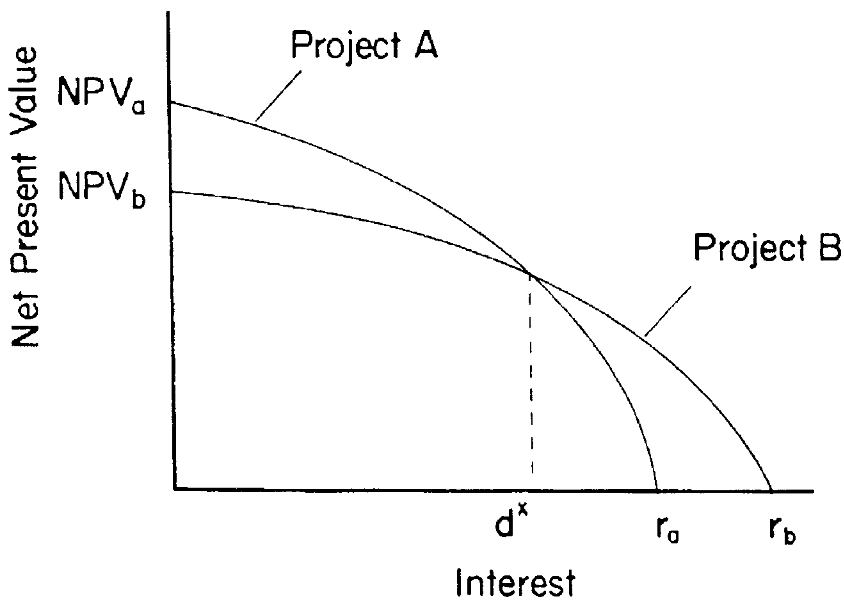

Figure 1 shows the *NPVs* of Projects A and B on the vertical scale, at various rates of interest shown on the horizontal scale. At zero interest, the *NPV* of Project A is higher than that of Project B, and both *NPVs* fall as interest rates increase. However, Project A is more affected by high interest rates than is Project B, either because the benefits from Project A occur later than those from Project B, or because the costs from Project A occur earlier. At d^* they have the same *NPV*. If the given rate of interest d is less than d^*, then Project A is preferred, but at interest rates greater than d^*, Project B is preferred. The interest rates that reduce the NPVs to zero are the internal rates of return, r_a and r_b, with r_b being higher. Both projects are viable, but Project B is preferable because it offers the higher IRR (r_b). The *NPV* method prefers Project A at any interest rate lower than d^*, but prefers Project B if the interest rate is greater than d^*. If one can earn r_b on Project B, that is the most plausible opportunity cost of capital to use in evaluating Project A.

The two methods are not strictly comparable. The IRR is independent of a project's monetary dimension, so projects of different sizes are not difficult to compare. By contrast, NPV is a sum expressed in money, and hence a larger project may have a larger NPV, even when the return on capital is lower.

DISADVANTAGES

Before further arguing the advantages of IRR, we must deal with a red herring. Because the calculation of the IRR involves solving a polynomial, critics (e.g., Drummond, 1980) sometimes argue that there can be more than one real solution to the equation, and therefore that the IRR is ambiguous. The correct response to this is that a second positive root only arises if there is a substantial negative cash flow after the project has been giving positive returns.

For example, consider a project that initially costs $100 and earns $350 at the end of the year, but must provide $300 at the end of the second year (for site restoration):

t_0, the beginning of the project — initial capital	$ -100
t_1, end of first year — net operating profit	$ +350
t_2, restoration at conclusion of project at end of second year	$ -300

In this example, the positive values for the IRR are either 50% or 100%.[4] This is equivalent to either (a) treating $50 of the earnings at t_1 as return on capital and $100 as capital repayment, leaving $200, which at 50% will produce $300 for the final payment, or (b) treating $100 as earnings and $100 as capital repayment, leaving $150, which at 100% will produce the final $300. Both results depend on the rate of return achievable on the sum reinvested to produce the final $300. This suggests the need for a sinking fund.

SINKING FUNDS

A sensible way of dealing with a substantial liability at the end of the project is to provide a sinking fund (Sugden & Williams, 1978). Money from the positive cash flow preceding the restoration is put aside to meet the estimated liability. This cash may be invested at the best rate of interest obtainable, from zero to the IRR of the project.

Of course, if it is not possible to find an investment return of 100% or 50% for a sinking fund, more money would have to be set aside, reducing the ultimate return on capital. Clearly, the rate of reinvestment is crucial; the sinking fund approach is useful in concentrating attention to this.

THE WEAKNESS OF THE NPV METHOD

The critical problem with the NPV method of evaluation is the choice of the appropriate discount rate. "Too low" a rate will bias comparison toward capital-intensive projects and toward longer term projects. "Too high" a rate has the reverse effect. The problem is to decide what constitutes "too high" and "too low." Of course, this problem does not arise with the IRR method, because it generates its own solution. In fact, the correct discount rate to use is the highest *internal* rate of return available on an alternative project.

In this context, there are some differences between the public and the private sector. A private profit-seeking firm has two possible approaches to the choice of discount rate. If the money to finance the project must be borrowed, then the correct rate would be the borrowing rate, but if the funds come from a firm's own resources, the correct rate is the best one that could be had if the money was invested on the market. This would be a slightly lower rate. By contrast, the real cost to society of undertaking a particular project, X, is the return that could have been had from the next most attractive project, Y, which would have been undertaken if X had not been chosen. This is what economists call the *opportunity cost.*

OPPORTUNITY COSTS

The opportunity cost of using capital funds in the public sector is not necessarily given by the market. The return we need is a measure of the *marginal productivity of capital* (the rate of increase in production arising from an extra unit of capital) in the public sector. This is the *IRR* of the next best project.

A complication arises where inflation is involved, as it inevitably is in long-term projects. The usual treatment is to do the whole calculation for both costs and benefits in terms of constant prices at a suitable base date. This is equivalent to assuming that all costs and all benefits will move with inflation. If there is reason to believe that a particular cost will rise relative to other costs this must be explicitly allowed for in the calculation. A case in point might be rising oil costs, due to the exhaustion of cheaper sources of energy. When this occurs, neither the market rate for borrowing nor the market rate for lending can be used without adjustment.

The adjustment is necessary because market rates of interest contain an allowance for the expected rate of inflation. If $100 is lent for a year at 5% interest, but at the end of that time there has been 6% inflation, the lender has lost out because the $105 will not buy as much as $100 a year ago. The lender has in fact suffered negative interest of 1%. To get a real return of 5% after inflation, he/she needs $111.30 ($105 x 1.06), or a nominal interest rate of 11.3%. Since the market allows for expected inflation, the market rate cannot be used in a calculation done at base-year prices, but it is no easy matter to decide how much inflation the market is anticipating.

CONCLUSIONS

Since the calculation of the IRR on all public projects would provide useful information to policy makers, it is surprising that it is not routinely provided. Part of the explanation, no doubt, lies in the tedious arithmetic that it previously entailed. In most cases, the only possible solution involved successive approximation, and these had to be repeated every time alternative assumptions regarding costs and benefits were tested. Recent developments in interactive computers completely change this picture, by reducing the calculation to a routine operation.

If information on the IRR were supplied on all public projects, it would be possible to compare the returns in the various subdivisions of the public sector on an objective basis, and perhaps to move toward more rational practices in the field of public capital expenditure. Using too low an interest rate for project evaluation with the NPV method, in connection with a system of budget allocations, is likely to lead to some departments proceeding with projects that are barely profitable, while other departments cannot undertake much more attractive projects because their capital allocations are exhausted.

There will still be a need for judgement in the final selection of projects and, in particular, in the weight to be given to nonquantifiable factors. From the point of view of those responsible for getting the best return on projects, subject to a budget constraint, calculation of the *IRR* on their projects could provide useful ammunition when arguing for future budget allocations.

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NOTES

1. If i is the annual rate of interest but interest is paid in half-yearly or more frequent installments, some of it is available sooner and hence begins to earn interest sooner:

$$P\ (1 + i/m)^{mn} = A_n$$

where m is the number of times per year the interest is added. The limit of this, as m gets very large, is:

$$Pe^{in}$$

where e is Euler's natural number ($e = 2.7183$, to 4 decimal places).

The significance of this is that the theoretical limit of continuous reinvestment of interest lies at the point where a firm receives continuous benefit from an investment, and uses the proceeds in the business as they become available.

2. At one time it was common to value a capital asset in terms of "years purchase." This is the number of years by which the annual net earnings must be multiplied in order to repay the initial outlay. For example, a business that would produce a clear annual profit of $500 and that is valued at 20 years purchase would sell for $500 x 20 = $10,000. The capital cost would be recovered over 20 years; once this was achieved, any further incremental earnings provide the reward that makes the investment attractive. Such transactions do not explicitly involve interest, although it is

possible (making some assumptions about the economic life of the asset) to calculate the implied rate of return (r) as follows:

$$K = \sum_{i=1}^{n} \left[\frac{a_i}{(1+r)^i} \right]$$

3. The Foster - Beesley (1963) evaluation of the Victoria Line in London was probably the first case where a full cost-benefit study was undertaken *before* the project was begun. The study showed that public investment, which was not financially viable, was justified on social grounds. The line was built on the strength of this study but, as J. M. Thomson (1974) has pointed out, encountered problems because no provision was made for the revenue subsidy necessary to pay for the social benefits.

4. The equation reducing cost and benefits to zero is:

$$- \frac{100}{(1+r)^0} + \frac{350}{(1+r)^1} - \frac{300}{(1+r)^2}$$

Let $1 + r = x$; then the equation is:

$x^2 - 3.5x + 3 = 0$

$x = 1.5$ or 2.0

But $r = x - 1$. Therefore $r = .5$ or 1.0

$r = 50\%$ or 100%